Vegetated Open Channel Design

A vegetated open channel (swale) was designed as requested to collect runoff adjacent to Smoketown Road in Bucknell University, before diverting though a culvert to discharge into Miller Run. The design criteria and design specifications have been summarized below:

Design Criteria:
- Vegetated open channel to collect runoff
- Channel should slow storm runoff to allow for more infiltration
- Maintain vegetation with a retardance class of C or D
- Assume a good stand of grass mixture (6 – 8 inches uncut)

Design Specifications:
- Design bed slop to follow the slope of the roadway.
- Design a cross section for the vegetated swale that satisfies the requirements for the entire length of the channel.
- Maintain at least a 3 ft shoulder.

The channel was analyzed using the 7 step process from *Hydraulic Engineering Circular No. 15, Third Edition* (HEC-15). Step 1 (determine discharge, channel slope, and shape) was done in accordance to *Pennsylvania Stormwater Best Management Practices Manual.* In Step 2, the lining type was used to find the permissible shear stress and Manning's n coefficient. Steps 3 through 5 involved estimating the channel depth and hydraulic radius to compute the discharge and iterate until the discharge was within 5 % of the design discharge. Steps 6 and 7 were to find the shear stress and compare it to the permissible shear stress. The analysis is explained in *Appendix 1: Steps 1 through 7* (pages 3 to 5). The final design of the swale obtained from this process is a trapezoidal channel with the following dimensions:

Bottom Width: b = 6 feet
Side Slopes: H:V = 3:1
Height with freeboard: 1.3 feet
Topwidth: T = 13.8 feet

The design is shown in Figure 1, obtained from the sketch to-scale in the Appendix. See Table 1 in Appendix 1 for recommended grass-legume mixtures to be used.

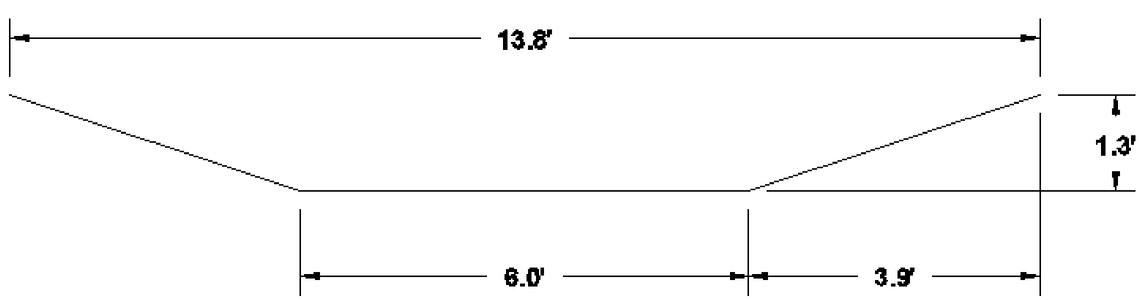

Figure 1: Vegetated Open Channel Design Cross-section (not to scale)

This trapezoidal channel is to be constructed at a distance of 3 feet (shoulder-road) from the road. Whilst the topwitdh of the channel was minimized, excavation of the hillside adjacent to the road (current profile) will be necessary to fit the channel. Figures 2 and 3 show the current profile and the channel profile; providing an overview of the area that will need to be excavated. A side slope of 3:1 was chosen to minimize the area that needs to be excavated.

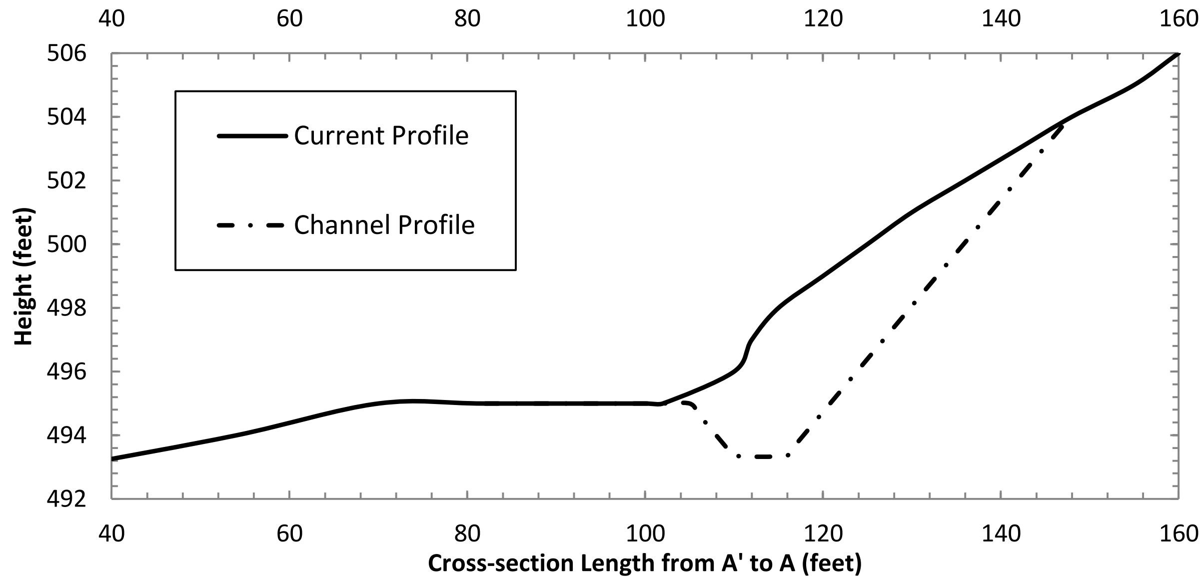

Figure 2**: Current Profile and Channel Profile at Cross-section A' to A

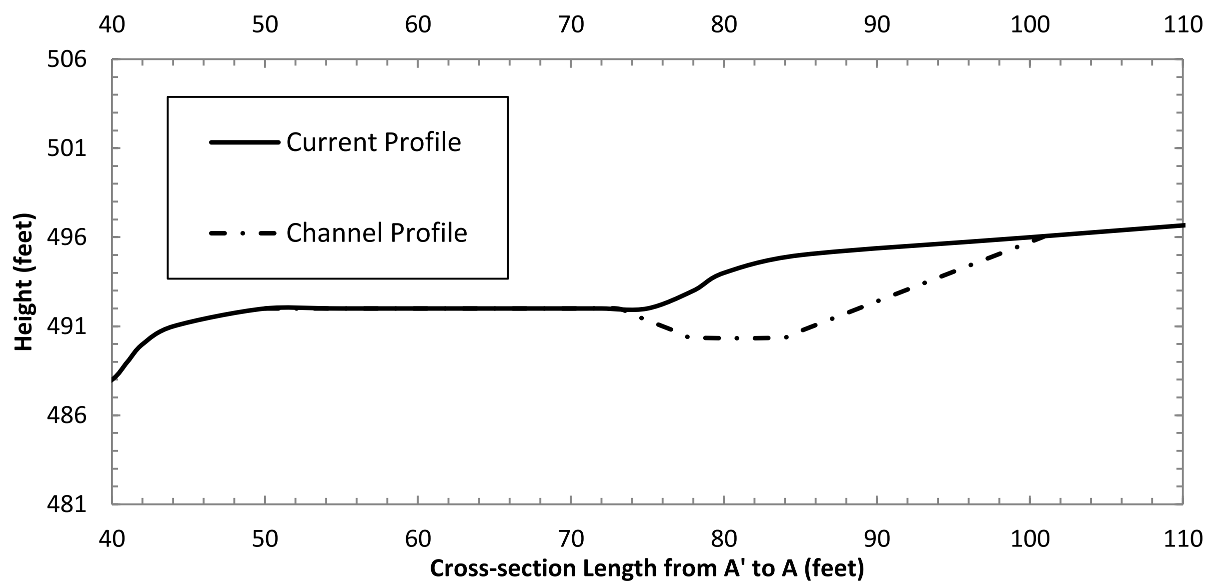

Figure 3**: Current Profile and Channel Profile at Cross-section B' to B

***Note: horizontal scale does not match vertical scale.*

Appendix 1: Steps 1 through 7

Step 1: Determine a design discharge and select the channel slope and channel shape

According to Chapter 6 of the *Pennsylvania Stormwater Best Management Practices Manual* (BMP), *6.4.8: Vegetated Swale*, the design discharge for the swale corresponds to the 10-year storm event. Other relevant information include a minimum infiltration rate of 0.5 inches per hour, which is assumed satisfied; side slopes ranging from 3:1 to 5:1 and a bottom width of 2 to 8 feet. The discharge, slope and selected design values are specified here. For calculations of the bed slope, please refer to the Appendix:

Design Discharge: $Q_{10} = 6.25\ cfs$
Channel Slope: $S_{0\ ,min} = 0.0080$
$S_{0,max} = 0.0125$
Channel Shape: Trapezoidal with side slope 3:1
Bottom Width: 6 feet

Step 2: Select Lining Type, Manning's Coefficient Values

Pennsylvania's climate accepts vegetation with retardance class C or D. From *Hydraulic Engineering Circular No. 15, Third Edition* (HEC-15): *Design of Roadside Channels with Flexible Linings*, the following permissible shear stresses and recommended seed-mixes are specified for uncut grass mixture (6 - 8 in.):

Table 1: Protective Cover Permissible Shear Stress & Recommended Cover

Retardance Class C Vegetation	1.0 lb./sq.ft.
Retardance Class D Vegetation	0.6 lb./sq.ft.
Recommended Cover: Grass-legume mixtures (C & D) including	
- summer (orchard grass redtop, Italian ryegrass, and common lespedeza) - fall, spring (orchard grass, Italian ryegrass, and common lespedeza) - Centipedegrass - Buffalo grass	

The empirical hydraulic roughness values for grass lining of 2 to 6 in. and 6 to 12 in. were retrieved from HEC-15. The maximum values correspond to flow velocities of 2 ft/s or lower, which is true in this design (See Tables 4, 6). The design grass has a length of 6 to 8 in., so the maximum values were interpolated for a length of 8 in. "Max. 8 inch", assuming the upper values represent the upper bound length of grass.

Table 2: Hydraulic Roughness (Manning's n) of Swales with Vegetation

Depth of flow	Channel	Minimum	Maximum	Max. 8 inch
Up to 0.7 ft.	Grass Length 2 in - 6 in	0.05	0.09	0.12
	Grass length 6 in - 12 in	0.08	0.18	
0.7 ft. to 1.5 ft.	Grass Length 2 in - 6 in	0.04	0.06	0.08
	Grass length 6 in - 12 in	0.06	0.12	

note: Higher Manning's n represents flow velocities of 2 ft/s or slower

Steps 3 through 5: Estimate channel depth and Manning's coefficient. Compute area, perimeter, hydraulic radius, and discharge.

The downstream portion of the swale is analyzed first. Design dimensions are below. "Max. 8 inch" is referred to as "Max. Manning's n" and depends on the depth of flow, in feet (See *Note in Appendix.):

Table 3: Downstream Channel Swale, Steeper Bed Slope

Side Slope m	3	d < 0.7 Max. Manning's n	0.12
Bottom Width b (ft)	6	d > 0.7 Max. Manning's n	0.08
Max. Slope S_0	0.0125	Design Discharge Q (cfs)	6.25

An initial depth d_1 = 0.62 ft. was guessed and the area, perimeter, hydraulic radius, and discharge were computed using equations 1 through 4 in Appendix 2. New depths were computed using equation 5 and the process was repeated until the discharge (Q_i) was within 5 % of the design discharge.

Table 4: Computing Discharge Q using Manning's Equation

Trial i	Depth of Flow d_i (ft)	Area A_i (ft^2)	Perimeter P_i (ft)	Hydraulic Radius R_i (ft)	Manning's n	Discharge Q_i (cfs)	Velocity V_i (ft/s)
1	0.620	4.87	9.92	0.49	0.10	5.05	1.04
2	0.675	5.42	10.27	0.53	0.10	5.89	1.09
3	0.691	5.58	10.37	0.54	0.10	6.15	1.10
4	**0.696**	5.63	10.40	0.54	0.10	6.22	1.11

The resulting shear stress is 0.543 psf., less than the permissible shear stress of 0.60 psf. for class D vegetation, meaning acceptable lining. The upstream (milder) portion of the swale was analyzed as well:

Table 5: Upstream Channel Swale, Milder Bed Slope

Side Slope m	3	d < 0.7 Max. Manning's n	0.12
Bottom Width b (ft)	6	d > 0.7 Max. Manning's n	0.08
Max. Slope S_0	0.008	Design Discharge Q (cfs)	6.25

Table 6: Computing Discharge Q using Manning's Equation

Trial i	Depth of Flow d_i (ft)	Area A_i (ft^2)	Perimeter P_i (ft)	Hydraulic Radius R_i (ft)	Manning's n	Discharg e Q_i (cfs)	Velocity V_i (ft/s)
1	0.700	5.67	10.43	0.54	0.10	5.03	0.89
2	0.763	6.33	10.83	0.58	0.10	5.89	0.93
3	0.781	6.52	10.94	0.60	0.10	6.15	0.94
4	**0.786**	6.57	10.97	0.60	0.10	6.22	0.95

The resulting shear stress is 0.393 psf., less than the permissible shear stress of 0.60 psf. The design dimensions are good for both the upstream (mild) and downstream (steeper) portions of the swale.

Step 6 and 7: Check for Shear Stress

Using the chosen design dimensions, the depths (Steps 3 through 6) and shear stress (Equation 6 in Appendix 2) were computed for Manning's values of 0.09 and 0.12 to verify that the design still fits the criteria.

Table 7: Range of Downstream and Upstream Depths of Flow and Shear Stresses

Downstream Channel Swale, Steeper Slope				Average	Standard Deviation
Manning's n	0.08	0.10	0.12	0.10	0.02
Depth of Flow (ft)	0.616	0.696	0.767	0.693	0.076
Shear Stress (psf)	0.481	0.543	0.599	0.541	0.059
Upstream Channel Swale, Milder Slope				**Average**	**Standard Deviation**
Depth of Flow (ft)	0.697	0.788	0.870	0.785	0.087
Shear Stress (psf)	0.348	0.393	0.434	0.392	0.043

The average shear stress for the downstream portion is 0.541 ± 0.059 psf, higher than upstream and thus controls the design. Assuming this follows a normal distribution, there's only a 16 percent probability of the shear stress being greater than the permissible shear stress (equation 7, Appendix 2).

The milder slope controls the depth needed, which is 0.785 ± 0.087 ft. The average is above 0.7 ft. so Manning's n would likely tend towards 0.8, leading to slightly shallower depth, not shallower than 0.7 ft., as this would lead to an increase in n. A depth of 0.80 feet is a reasonable estimate, and hence it is chosen as the design flow depth. The necessary overboard is 6 inches or 0.5 feet according to BMP, thus the total depth is 1.3 feet. The results of this analysis lead to the dimensions specified in Page 1.

Appendix 2: Equations and Sample Calculations

Step 1: Determining the Slope of the Open Channel.
In Map provided: red contour to red contour = 5 feet difference in elevation
Yellow contour to yellow contour = 1 feet difference in elevation

Table 8: Determining the slope of the channel along the Smoketown road

Channel Distance, L (ft.)	Elevation H (ft.)	Slope ($\Delta H/\Delta L$)
0	-	-
10	497	0.0080
135	496	0.0087
250	495	0.0125
330	494	0.0125
410	493	-

(Note: L= 0 ft. represents the starting point of the channel shown in the map)

Sample calculation of slope: from distance 330 feet to 410 feet:

$$\frac{\Delta H}{\Delta L} = \frac{(494-493)ft}{(410-330)ft} = 0.0125$$

Step 2: Interpolation for Manning's n coefficients "Max. 8 inch"
Assuming that 0.18 corresponds to grass of length 12 in., and 0.09 corresponds to grass of length 6 in. and using a linear interpolation:

$$Max. 8\ inch\ (d < 0.7\ ft.) = 0.09 + \frac{0.18 - 0.09}{12\ in. - 6\ in.} * (8in. - 6\ in.) = 0.12$$

Assuming that 0.12 corresponds to grass of length 12 in., and 0.06 corresponds to grass of length 6 in. and using a linear interpolation:

$$Max. 8\ inch\ (0.7\ ft. < d < 1.5\ ft.) = 0.06 + \frac{0.12 - 0.06}{12\ in. - 6\ in.} * (8in. - 6\ in.) = 0.08$$

(Note: The values represent rough estimates of the actual Manning's coefficient)

Steps 3 through 6: Equations used to Compute area A_i, wetted perimeter P_i, hydraulic radius R_i, discharge Q_i and new depth d_{i+1} for each trial i :

$$A_i = b * d_i + m * d_i^2 \quad (1)$$

$$P_i = b + 2d_i\sqrt{1 + m^2} \quad (2)$$

$$R_i = {}^{A_i}/_{P_i} \quad (3)$$

$$Q_i = \frac{k_m}{n} * A_i * R_i^{2/3} * S_0^{1/2} \quad (4)^*$$

$$d_{i+1} = d_i \left({}^{Q}/_{Q_i}\right)^{0.4} \quad (5)$$

*(*Note: For Manning's n, an averaged n of 0.10 was used to account for the fact that the flow depth was very close to the "transition" depth of 0.7 feet for most trial dimensions. The chosen dimensions were also checked using the Manning's values of 0.08 and 0.12 to find the range of acceptable depths by verifying the shear stress. Using values of 0.08 usually led to depths requiring 0.12 and vice-versa)*

Sample Calculations using Appendix 1, Table 4, Trial 3:

Eqn. 1→ $A_3 = b * d_3 + m * d_3^2 = 6\ ft. * 0.691\ ft. + 3 * (0.691\ ft.)^2 = 5.578\ ft^2$

Eqn. 2→$P_3 = b + 2d_3\sqrt{1 + m^2} = 6\ ft. + 2(0.691\ ft.)\sqrt{1 + 3^2} = 10.37\ ft.$

Eqn. 3→$R_3 = {}^{A_3}/_{P_3} = 5.578\ ft^2/10.37\ ft. = 0.538\ ft.$

Eqn. 4→$Q_3 = \frac{k_m}{n} * A_3 * R_3^{2/3} * S_0^{1/2} = \frac{1.49}{0.10}(5.578\ ft^2)(0.538)^{2/3}(0.0125)^{1/2} = 6.147\ cfs$

Eqn. 5→$d_4 = d_3\left({}^{Q}/_{Q_3}\right)^{0.4} = 0.691\ ft.\left({}^{6.25}/_{6.147}\right)^{0.4} = 0.696\ ft.$

Step 7: Calculating Shear Stress:

$$\tau_d = \gamma_w d S_0 \quad (6)$$

Where
τ_d ≡ shear stress
γ_w ≡ unit weight of water = 62.4 psf
d ≡ final depth
S_0 ≡ bed slope

Sample Calculation of Shear Stress: Using depth from Appendix 1, Table 4, Trial 4:

Eqn. 6 → $\tau_d = \gamma_w d S_0 = 62.4\, psf * 0.696\, ft * \ 0.0125 \ = \ 0.543\, psf$.

Is $0.543\, psf < 0.60\, psf$? Yes → Channel dimensions and lining are Ok.

$$P(x < 0.6) = \int_{-inf}^{0.6} \frac{1}{\sigma\sqrt{2\pi}} \mathrm{EXP}\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right) dx \quad (7)$$

Where
x ≡ shear stress; $-inf < x < 0.6$ psf.
μ ≡ mean of shear stress = 0.55 psf.
σ ≡ standard deviation of shear stress = 0.06 psf.

$$P(x < 0.6) = \int_{-inf}^{0.6} \frac{1}{0.06\sqrt{2\pi}} \mathrm{EXP}\left(-\frac{1}{2}\left(\frac{x-0.55}{0.06}\right)^2\right) dx = 0.841$$

i.e. there's 84 % the shear stress is less than 0.60 psf, or 16 % chance the shear stress exceeds 0.60 psf. Overall, the probability of failure of the channel swale is $P(fail) = P(Q > Q_{10})P(\tau > 0.60) = 0.10 * 0.16 = 0.016$ or 1.6 percent.

Step 8: Determining the Elevation at different points of the Cross sections A-A' and B-B'

Left columns represent current profile and right columns represent channel cross-section.

Table 9: Horizontal Distance and Elevations for Current Profile & Channel Profile at Cross-sections A'-A and B'-B

Section A'-A		Section A'-A	
Horizontal Distance (ft.)	Elevation (ft.)	Horizontal Distance (ft.)	Elevation (ft.)
0	488.5	**82**	**495**
3	489	**100**	**495**
6	490	**102**	**495**
12	491	105.0	495.0
20	492	106.0	494.7
35	493	107.0	494.3
54	494	108.0	494.0

Section B'-B		Section B'-B	
Horizontal Distance (ft.)	Elevation (ft.)	Horizontal Distance (ft.)	Elevation (ft.)
0	486.5	**50**	**492**
13	484	**55**	**492**
28	485	**70**	**492**
36	486	73	492
38	487	74.0	491.7
40	488	75.0	491.3
41	489	76.0	491.0

70	495	109.0	493.7
82	**495**	109.9	493.4
100	**495**	115.9	493.4
102	**495**	116.9	493.7
110	496	117.9	494.0
112	497	118.9	494.4
115	498	119.9	494.7
120	499	120.9	495.0
125	500	121.9	495.4
130	501	122.9	495.7
136	502	123.9	496.0
142	503	124.9	496.4
148	504	125.9	496.7
155	505	126.9	497.0
160	506	127.9	497.4
168	507	128.9	497.7
172	508	129.9	498.0
180	509	130.9	498.4
185	510	131.9	498.7
195	511	132.9	499.0
201	512	133.9	499.4
210	513	134.9	499.7
220	514	135.9	500.0
230	515	136.9	500.4
239	515.8	137.9	500.7
		138.9	501.0
		139.9	501.4
		140.9	501.7
		141.9	502.0
		142.9	502.4
		143.9	502.7
		144.9	503.0
		145.9	503.4
		146.9	503.7
		147.9	504.0

42	490	77.0	490.7
44	491	77.9	490.4
50	**492**	83.9	490.4
55	**492**	84.9	490.7
70	**492**	85.9	491.0
72	492	86.9	491.4
75	492	87.9	491.7
78	493	88.9	492.0
80	494	89.9	492.4
85	495	90.9	492.7
100	496	91.9	493.0
115	497	92.9	493.4
121	498	93.9	493.7
127	499	94.9	494.0
135	500	95.9	494.4
142	501	96.9	494.7
150	502	97.9	495.0
158	503	98.9	495.4
165	504	99.9	495.7
170	504.8	100.9	496.0

(Note: The bold data in the tables represent the section where Smoketown Road lies)